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Research Article

Applying a triple logistic growth model to *Penaeus vannamei* farmed in earthen ponds at a commercial farm in the Gulf of California, Mexico

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ABSTRACT. Shrimp farmed for a long period pauses its growth because of changes in environmental conditions or the achievement of the pond's carrying capacity. A modification of the von Bertalanffy model has been suggested to describe seasonal or oscillatory growth. Multiple logistic function is another alternative. This study used data from three earthen ponds farming the white legs shrimp *Penaeus vannamei* from spring to autumn, 202 days of commercial culture. An oscillatory version of the von Bertalanffy model, the Richards model, and double and triple logistic functions were contrasted. The best model was selected using information theory based on the Bayesian information criterion (BIC). We assumed a normal distribution of error. The triple logistic was the best model to describe the growth in the three earthen ponds because the lowest BIC value and BIC weight is more than 90%. The present study concluded that the multi-logistics approach best describes the growth in farmed shrimp *P. vannamei* if partial harvest is applied.

Keywords: *Penaeus vannamei*; aquaculture; information theory; von Bertalanffy; multistage growth

INTRODUCTION

The Pacific whiteleg shrimp *Penaeus vannamei* Boone, 1931, is native to the western tropical coast of the American mainland. However, this species is a very important aquaculture target worldwide. Seventy percent of global farmed shrimp production is obtained from this species, even in countries without its natural distribution (FAO 2024). In Latin America, particularly in Mexico, nearly 100% of the farmed shrimp consists of *P. vannamei*. In 2021, the total harvest from aquaculture activities of *P. vannamei* was 182,110 mt.

Using mathematical equations to describe the growth in farmed shrimp is not common; instead, it is usually thought that raised shrimp display linear growth. Therefore, the most common practice is the specific growth rate that estimates the final size (weight or length) minus the initial size divided by the culture period. Even in this routine, some studies tried different mathematical equations to describe growth in farm-raised shrimp (Yu et al. 2006, Aragón-Noriega 2016, Castillo-Vargasmachuca et al. 2021). It is also important to consider that bioeconomic analysis in aquaculture activities has gained popularity in projecting better cost analysis and production optimization (Hernandez-Llamas et al. 2004). Bioeconomic studies will require a production model that necessarily implies modeling the species' growth in culture. For this reason, modeling growth in species under aquaculture conditions is also achieving acceptance among the academic community.

Before *a priori* selection of a growth model, it is important to consider the seasonal oscillations of growth, also known as growth pauses, due to environ-

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mental or physiological stress factors. The observed pause in the growth of the individual has been identified as being caused by numerous situations, such as changes in temperature and salinity due to rainy and dry seasons because species in commercial aquaculture go through several seasons of the year. Other characteristics are changes in the types or preferences in feeding and the carrying capacity of the cultivation ponds due to the increase in biomass, among others. Multi-logistic models such as double or triple logistic models (Modis 1994, Shabani et al. 2018) are options for this feature. This approach enhances a single logistic function. It is a stage with two phases with an inflection point. The growth rate increases before the inflection points and decreases afterward. Double logistics are two stages with two phases each; the same process is applied for the growth rate before and after each inflection point. This process continues as many logistics could be used in the study. Modis (1994) proposed a logistics study for the human population, but similar growth was also observed in the growth of agricultural crop species (Shabani et al. 2018). If two or more sigmoid curves are used to describe a phenomenon, a logistic function should not be used. Instead, the researcher may apply two Gompertz functions or a combination of the logistic function and Gompertz functions (Hau et al. 1993).

Weight-at-age data are more commonly used than length-at-age data in raised shrimp. Generally, the weight-at-age data describe a sigmoid-shaped growth curve. The growth rate reaches a maximum, corresponding to an inflection point in the curve, and then slowly declines to zero when the animals achieve their mature weight. In the present study, the objective was to use data from three earthen ponds to describe the growth of white legs shrimp *P. vannamei*, cultured for 202 days covering a seasonal period from spring to autumn to contrast the oscillatory von Bertalanffy growth model, Richards model (Richards 1959), double, and triple logistic models to describe individual growth in weight.

MATERIALS AND METHODS

This study was carried out in a commercial farm dedicated to the aquaculture of white shrimp *P. vannamei*, located in the southern part of the Altata-Ensenada del Pabellón estuarine lagoon system, 40 km west of Culiacan, Sinaloa, Mexico (24°22"12'-24°19"56'N and 107°29"10'-107°27"17'W). The lagoon is in the mouth of the Gulf of California. The three earthen ponds used belong to "Sociedad Cooperativa de Producción Pesquera Ejidal El Patagüe, S. de R.L. de C.V.", which is supplied with seawater through the intake channel from the sea.

The shrimp culture was evaluated from April 27 to November 15, 2023, in three earthen ponds with bottom areas of 12 ha, stoked at 15 postlarvae (PL) m^{-2}. The PL of white shrimp *P. vannamei* were obtained from a local commercial hatchery at size PL12. Biometric sampling of shrimps was carried out weekly in each of the ponds studied, using cast nets with different mesh sizes depending on the size of the organisms during the production cycle. One hundred specimens were randomly selected and weighed individually using an Ohaus brand digital scale, with reading precision adjustable to 0.1 and 0.01 g, according to the size of the organisms.

This study focuses on modeling growth through new approaches to multi-logistic equations. First, double and triple logistic models were selected, considering more than one stage, and each stage considers two phases with an inflection point. Two very well-known models were also used. Richard's generalized growth model (Richards 1959) has a dimensionless parameter that lets the curve follow different trajectories. The von Bertalanffy with an oscillatory component (VBO) provides the easiest visualization of seasonal growth. The models used to describe the growth of *P. vannamei* are:

von Bertalanffy oscillatory model:

$$Yt = Y_\infty \left(1 - e^{-\left[c\, sin\, 2\pi\frac{(t-ts)}{250} + k(t-to)\right]}\right) \quad (1)$$

Richards model (Richards 1959):

$$Yt = Y_\infty (1 - e^{-k(1-m)(t-t_o)})^{(1/1-k)} \quad (2)$$

Double logistics model:

$$Yt = \frac{Y_{\infty 1}}{(1+e^{-k1(t-t0_1)})} + \frac{(Y_{\infty 2}-Y_{\infty 1})}{(1+e^{-k2(t-t0_2)})} \quad (3)$$

Triple logistics model:

$$Yt = \frac{Y_{\infty 1}}{(1+e^{-k1(t-t0_1)})} + \frac{(Y_{\infty 2}-Y_{\infty 1})}{(1+e^{-k2(t-t0_2)})} + \frac{(Y_{\infty 3}-Y_{\infty 2})}{(1+e^{-k3(t-to3)})} \quad (4)$$

where *Yt* is the weight at age *t*. Y_∞, $Y_{\infty 1}$, $Y_{\infty 2}$, and $Y_{\infty 3}$ are the sizes at which growth slows down (asymptotic weights). k, k_1, k_2, and k_3 are growth coefficients with units of t^{-1}. t_o is an adjustment parameter of the von Bertalanffy model that represents the theoretical age when the length is 0. t_{01}, t_{02}, and t_{03} are the inflection ages of the growth curve in the Logistic models.

The maximum log-likelihood *LL* functions were used to parametrize the models, assuming a normal distribution of the errors. That means the optimum parameters were selected when the maximum *LL* was reached.

$$LL = \left(\frac{-n}{2}\right)\left[Ln\ (2\pi) + 2Ln\left(\sqrt{\frac{\sum(Y_t-\widehat{Y_t})^2}{n}}\right) + 1\right] \quad (5)$$

where Y_t is the observed length or weight at age *t*, ($\widehat{Y_t}$) is the estimated length or weight with any candidate model, and *n* is the total observations.

The Bayesian information criterion (BIC) was estimated as follows to select the best model:

$$BIC = -2LL + Ln(n)\theta$$

where *LL* is the log-likelihood value obtained by normal functions, θ is the number of parameters in each model, and *n* is the number of paired size-at-age data used. The model with the lowest value was selected as the best. The hierarchical order for the model was obtained by differences in BIC values as follows:

$$\Delta_i = BIC_i - BIC_{min}$$

The obvious result is that $\Delta_i = 0$ is the best model. Another factor to be considered is the weight of evidence in favor of the model. The BIC weight (W_i) was estimated as described by Burnham & Anderson (2002) using the following formula:

$$W_i = \frac{e^{(-0.5\Delta_i)}}{\sum_{i=1}^{4} e^{(-0.5\Delta_i)}} \quad (6)$$

RESULTS

The total culture period was 202 days, covering the climatic seasons of spring, summer, and autumn. The exact dates were April 27 to November 15, 2023. As growth paused, the strategy was to carry out a partial harvest. They were done on August 2 (97 days of culture), August 24 (119 days of culture), and October 5 (161 days of culture) (Fig. 1).

The best model to describe the growth pattern of *P. vannamei* in the three commercial earthen ponds was the triple logistic model, with a normal distribution in the error structure. It was thus concluded because it obtained the lowest BIC among the other candidate models (Table 1). The VBO model was positioned second even though the curve was far from the observed data (Figs. 2,4). Double logistics was the worst model because it obtained the highest BIC value. The parameters of the triple logistic model for each pond are shown in Table 2.

Production and environmental results were not the focus of the present study. Still, the salinity was at 36 at the beginning of the culture period, reaching its maximum of 42 in August and descending to 38 in November. The average water temperature in the pond was 27°C at the beginning of the culture period, reached its maximum in July (32.9°C), and dropped to 26°C at harvest. The yield was 1,528 kha^{-1} in pond 1, 1,633 kha^{-1} in pond 2, and 1,572 kha^{-1} in pond 3.

DISCUSSION

The improvements in growth models were constrained for many years because of the tremendous domination of the von Bertalanffy growth model, which almost exclusively selected *a priori* in stock assessments, even when better fitting models had been found for the most important species in commercial fisheries. Some attempts have been made to improve traditional modeling in aquaculture studies, such as those recommended for fishes, crustaceans, and mollusks by Hernandez-Llamas & Ratkowsky (2004). This model was later modified by Ruiz-Velazco et al. (2010) and applied to producing *P. vannamei* raised on three farms with intensive production technology. The two previous studies compared the performances of von Bertalanffy's growth model to the model they described as new. Rodríguez-Domínguez et al. (2024) proposed using a fractal approach to describe the totaba (*Totoaba macdonaldi*) larvae, Japanese oysters (*Magallana gigas*), and Pacific white shrimp. The multi-logistic approach applied in the present study may be an important improvement in the growth model performance. However, criticism could be expected because it is supposed that a more complex model will usually be selected over a simpler model. However, using an information-theoretic index to select a model such as BIC includes a factor that penalizes the number of parameters. In other words, the BIC considers model parsimony, such that models with fewer parameters will be selected. Flinn & Midway (2021) reviewed more than 300 papers on individual growth. They found that many authors have argued against the most complex model because they said the goodness of fit test very often selects the models with more parameters.

The present study selected triple logistic models as best in the three earthen ponds. However, the double logistic model was the worst according to BIC value, even though it was the second most complex model. This result suggests that BIC considered parsimony over complexity. Even though BIC also considered the realism to fit the observed data to a model because the parametrization was done using maximum likelihood. So, the best model was selected by combining maximum likelihood and penalization to model with more parameters. A good example is that the selected model should have appropriate statistical support and a

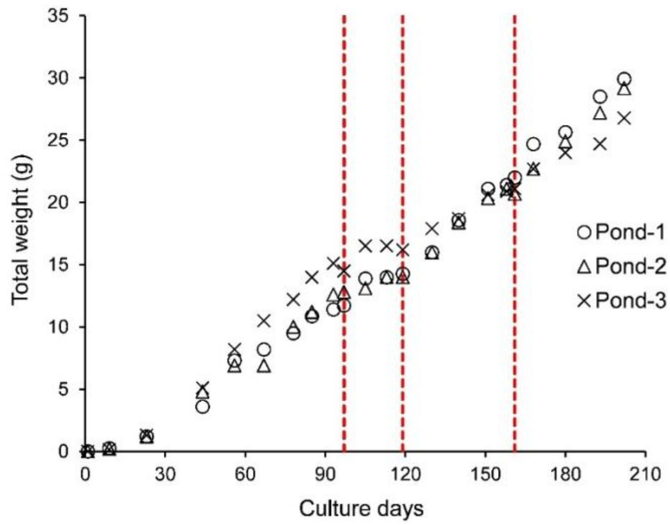

Figure 1. The observed data described *Penaeus vannamei* growth in earthen ponds on a commercial farm in the Gulf of California, Mexico. The dotted lines indicated a period of partial harvest.

Table 1. Bayesian information criterion (BIC) values are used as a goodness of fit in each candidate model to describe the growth of farmed *Penaeus vannamei*. W_i is the percent in favor of each model. Δ_i is the differences in BIC values, $\Delta_i = BIC_i - BIC_{min}$

Species	Model	Parameters	BIC	Δ_i	W_i (%)
Pond 1	Triple logistic	10	59.8	0.0	94
	von Bertalanffy	5	65.4	5.6	6
	Richards	5	73.8	14.0	0
	Double logistic	7	88.8	29.1	0
Pond 2	Triple logistic	10	59.4	0.0	99
	von Bertalanffy	5	66.0	6.6	1
	Richards	5	77.5	18.1	0
	Double logistic	7	90.7	31.3	0
Pond 3	Triple logistic	10	56.9	0.0	100
	von Bertalanffy	5	74.7	17.9	0
	Richards	5	83.0	26.2	0
	Double logistic	7	96.4	39.6	0

biologically good fit. For example, Aragón-Noriega (2016) found that the asymptotic model better describes the growth of *P. vannamei* farmed on earthen ponds, and Castillo-Vargasmachuca et al. (2021) arrived at the same result with the same species, but in lined ponds at an indoor facility. Those authors did not let the cultivation continue for longer periods. What does the above discussion mean? Most aquaculture studies in bivalves, crustaceans, and finfishes usually use the specific growth rate that implicitly says that a linear model may fit the data better over some cultivation periods. However, everybody knows that biologically, any farmed animal does not grow linearly, and instead, they will reach an asymptotic size because environmental or physiological stress will pause the growth.

In the present study, the three earthen ponds paused the growth at 97 days of the farmed period, similar to those reports by Aragón-Noriega (2016). In this case, the decision was to apply a partial harvest to defeat the stress factor. As a result, the growth rate did not exhibit a recuperation, and three weeks later, a second partial harvest was implemented, resulting in a new increasing growth rate (Fig. 1). Due to the successful growth rate recuperation a third partial harvest was applied at 161 days of farming. As seen in Figure 1, the growth was increasing significantly, but the final harvest was motivated by the commercial purpose of the farms. Therefore, the analyses here seem to correspond to three logistic harmonics represented in the triple logistic model, which was the best way to improve bio-

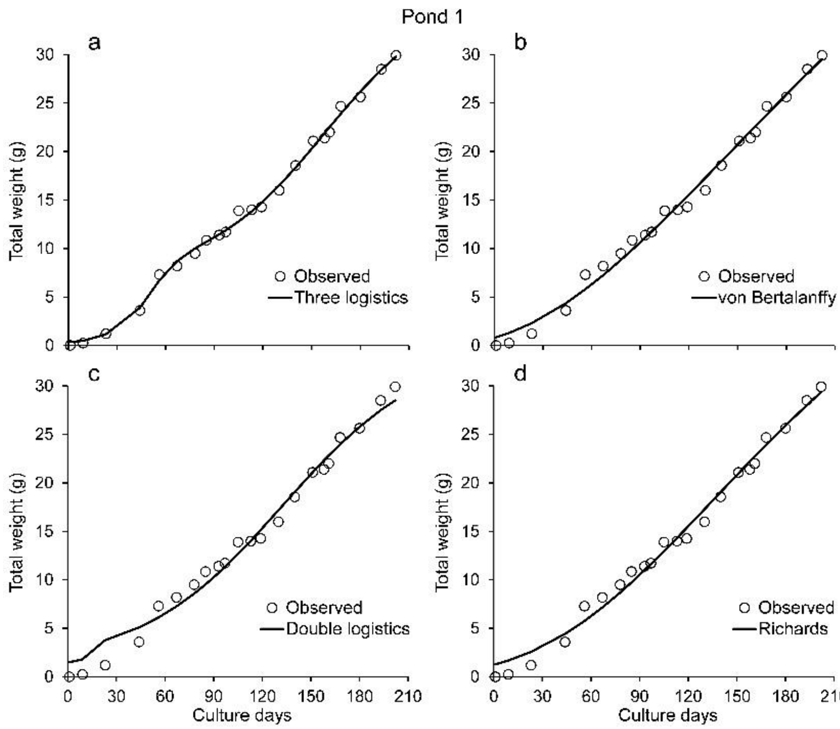

Figure 2. Trajectories of the four models used to describe the growth of the farmed white legs shrimp *Penaeus vannamei* in the earthen pond 1 on the farm located in the mouth of the Gulf of California, Mexico. a) Three logistics model, b) von Bertalanffy model, c) double logistics model and d) Richards model.

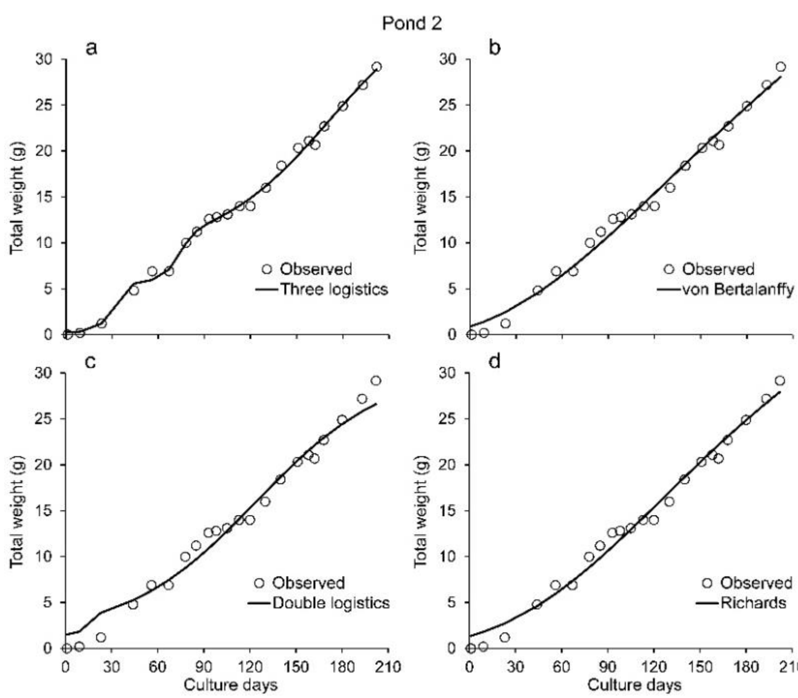

Figure 3. Trajectories of the four models used to describe the growth of the farmed white legs shrimp *Penaeus vannamei* in the earthen pond 2 on the farm located in the mouth of the Gulf of California, Mexico. a) Three logistics model, b) von Bertalanffy model, c) double logistics model and d) Richards model.

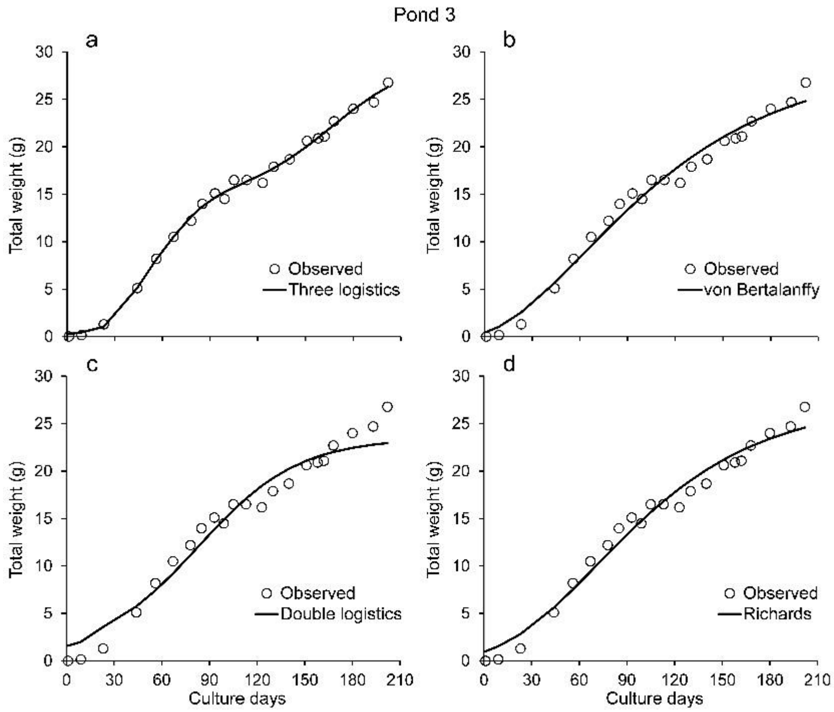

Figure 4. Trajectories of the four models used to describe the growth of the farmed white legs shrimp *Penaeus vannamei* in the earthen pond 3 on the farm located in the mouth of the Gulf of California, Mexico. a) Three logistics model, b) von Bertalanffy model, c) double logistics model and d) Richards model.

Table 2. Parameters, after fitting a triple logistic model to growth data of farmed *Penaeus vannamei* in the earthen ponds. $Y_{\infty 1}$, $Y_{\infty 2}$, and $Y_{\infty 3}$ are the sizes at which growth slows down (asymptotic weights). k_1, k_2, and k_3 are growth coefficients with units of t^{-1}. t_{01}, t_{02}, and t_{03} are the inflection ages of the growth curve in the logistic models.

Logistic harmonic	Parameters	Pond 1	Pond 2	Pond 3
	$Y_{\infty 1}$	0.18	2.36	4.7
1	k_1	2.66	2.11	2.83
	t_{01}	16	44	24
	$Y_{\infty 2}$	7.66	15.10	9.3
2	k_2	0.11	0.07	0.207
	t_{02}	49	60	74
	$Y_{\infty 3}$	36.02	29.19	37.4
3	k_3	0.03	0.04	0.027
	t_{03}	158	167	171

logical realism. The fitting triple logistic model represents a realistic performance of *P. vannamei* under the farmed condition in this facility. It should be recognized that the utility of the triple logistic function would be limited to the case of shrimp culture with three partial harvests. Caution is suggested to its application in future studies. The suggestion is that even if a model selection method suggests a simpler model over a more biologically relevant model, the researcher should consider the value of a better statistical fit *versus* a better biological fit that can approximate the actual growth trajectory. Researchers should not sacrifice biologically reasonable parameter estimates for improved statistical fit.

Credit author contribution

B. Ramos-Torres: conceptualization, validation, methodology, data curation, review, and editing; J.A. Félix-Ortiz: funding acquisition, project administration, supervision, review, and editing; R. Urías-Sotomayor: methodology, validation, supervision, review, and editing; G. Rodríguez-Domínguez: methodology, formal analysis, review, and editing; J. Payán-Alejo: methodology, formal analysis, review, and editing; E.A. Aragón-Noriega: conceptualization, validation, methodology, data curation, formal analysis, writing-original draft, funding acquisition, project administration, supervision, review, and editing.

Conflict of interest

The authors declare no potential conflict of interest in this manuscript.

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